

Low Volatility Asset Valuation in Brazilian Stock Market: Lower Risk with Higher Returns

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Abstract

This work evaluates the behavior of portfolios comprised of Brazilian stocks ranked by their volatility to investigate the low volatility anomaly.

Between January 2003 and December 2021, the low volatility portfolio presented a 6% annual return above the high volatility portfolio. This result is aligned with the observation made by Blitz and Van Vliet (2007) in global markets, with an annual alpha spread of 12% over the period between 1986 and 2006.

Also, through a double sorting process, it was possible to obtain portfolios with higher returns and lower risk than those ranked by a single risk factor, although this difference was not statistically significant in most cases.

Keywords: Low Volatility, Low Risk Stocks, Fama-French Factors, CAPM

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1. INTRODUCTION

Modern Portfolio Theory, developed by Harry Markowitz (1952) assumes that investors seek to maximize the Sharpe Ratio (SR), which is the maximum expected return for a level of risk. The model also assumes that these investors are risk-averse. Given two assets with the same expected return, investors, therefore, prefer the lower-risk asset. For this reason and assuming agents act rationally, a riskier asset increases the probability of higher returns.

In this paper, we study the risk-return relationship of portfolios created by arranging assets based on their realized volatility between 2003 and 2021, considering the stocks in the Brazil Index 100 (IBrX 100), a market index whose aim is to be a weighted performance indicator of the 100 stocks with the greatest representativeness and tradability in the Brazilian stock exchange. In this analysis, we found a negative relationship between risk, measured by the beta of each portfolio, and the return of each portfolio. For this analysis, 3 portfolios were formed. The first and last portfolios comprised 30% of assets at extremes (higher and lower volatility) and the remaining 40% of assets in an intermediate group. In a comparative analysis among these three larger groups, we had the larger Sharpe Ratio (SR) for the portfolio formed by lower volatility stocks. The results are consistent with those of Blitz and Van Vliet (2007) for the U.S., European, and Japanese stock markets using the same method.

In a 2013 study, Daniel and Moskowitz found that strategies that attempt to capture alphas from momentum portfolios can have extremely negative returns. We repeat this analysis for volatility portfolios to test whether the behavior is like those of momentum strategies. In 2009, the year after the 2008 global financial crisis, the return difference between the higher and the lower volatility was 116.2%. If the investor examines the persistent performance between the two portfolios observed from 2003 to 2008, the 2009 performance would be enough to wipe out the positive returns of the previous years, like the momentum strategy observed in Daniel and Moskowitz's 2013 study.

Subsequently, we evaluate portfolios using the CAPM model. We found that no portfolio generated an alpha that was statistically significant at the 5% significance level.

The same stocks were used to form portfolios based on other characteristics such as size, value, momentum, and profitability. We then reordered portfolios, depending on the volatility of each stock. This procedure resulted in portfolios with higher Sharpe ratios than the portfolios formed with only one factor.

Then, we developed an allocation strategy that exploited the observed behavior of the low volatility portfolio relative to the high volatility portfolio. The strategy generated an

annualized return of 14.95% with close to zero exposure to the market index. To construct a zero-beta portfolio, an ex-post beta model and an ex-ante beta model were used using the convergence proposed by Blume (1975).

Finally, we considered possible explanations for the observed risk-return behavior of portfolios sorted by volatility. We relate one of the likely explanations to the fact that agents consider CAPM model as an instrument capable of reflecting the asset returns expectations. We relate another hypothesis to the fact that investors seek assets with higher volatility in the expectation of greater returns as if it were a lottery ticket, and this behavior leads to an increase in the demand for this type of asset to the detriment of assets with lower volatility.

2. DATA AND METHODOLOGY

We developed the model using data from Bloomberg. Then, collected daily price data from January 2003 to December 2021. The stocks used for data collection were members of the IBrX 100 on the cut-off date of the particular month used in the model.

The IBrX 100 was chosen for its static composition, with 100 members with the highest tradability and representativeness in the Brazilian stock market. Therefore, the members are sufficiently liquid to build the portfolios in this analysis and replicate them systematically with sufficient liquidity. The index is rebalanced every four months.

We used long-term volatility (annual) to ensure fewer turnovers in each portfolio during the monthly rebalancing. Blitz and Van Vliet (2007) suggested an even longer time frame (3 years). Assuming a longer volatility window in the local market would significantly reduce the number of stocks with available information since most IBrX components were relatively new to the market during the period analyzed and few quantitative data, such as momentum and volatility itself, would be available.

Initially, we created three portfolios by ranking the volatility of IBrX members each month. We created the rankings from lowest to highest volatility, with the lowest volatility stocks forming the first portfolio and the highest volatility stocks forming the last portfolio. These portfolios were ranked and numbered from 1 to 3, the first portfolio called P1 and the last portfolio called P3.

We also formed ten portfolios from the risk factors of value, momentum, size, and profitability. We created the value portfolio from the analysis of the relationship between the book value of each company and its market value. The calculation methodology followed the one proposed by Asness and Frazzini (2013) and was called High Minus Low Devil (HML). In the model proposed by Fama & French (1993), the rebalancing occurred every six months,

although the market value of each company was available daily. In contrast, two methods are proposed for the Asness and Frazzini's factor. One method considered the most recent information, i.e., the market value using the last market price for each company and the book value reported in the last published earnings reports, re-weighting these portfolios monthly. Asness and Frazzini found that this method generated more alpha than the other proposed models when analyzing multiple stock markets worldwide.

The momentum portfolio construction strategy has been widely discussed since the study by Jegadeesh and Titman (1993), which evaluated 16 different momentum strategies. Rouwenhorst (1998) was the first to test this factor in Brazil, but he used only one strategy: 6 months of cumulative returns and semi-annual rebalancing. In our study, we also used 6-month total returns, but unlike Rouwenhorst, we chose a monthly rebalancing.

Next, portfolios were formed by size based on the market value of each asset, and finally, a profitability factor, that considers the ranking of assets by their gross profit to total assets ratio. This methodology follows the one of Novy Marx (2013), where the author shows that this metric has roughly the same explanatory power as book-to-market (value) in explaining the cross-section of expected returns.

As with the volatility portfolio construction, we ranked size in ascending ranking, i.e., the first decile had stocks with lower size and companies with higher market value to the last deciles. For momentum, value and profitability, we used descending ranking, i.e., stocks with higher observed values are assigned to the first deciles and vice versa.

Dividing the 100 components within IBrx into deciles creates portfolios of 10 stocks each, which should have higher idiosyncratic risk than the stock universe. Thus, larger groups of assets were created by aggregating some portfolios. This process of portfolio aggregation followed the logic of Fama & French's factorization process, i.e., the first low volatility consists of 30% lower volatility stocks, followed by an intermediate group of 40% and, finally, the remaining 30% that have higher volatility.

We determined the alpha of each of the portfolios sorted by volatility, using the CAPM and Fama-French three-factor model to evaluate their statistical significance. We also tested the same groups by adding a momentum factor, or Winners Minus Losers (WML), and a profitability factor, or Profitable Minus Unprofitable (PMU).

The WML, HML, and PMU factors are composed of the difference between the returns of the group containing the top 30% of assets and the bottom 30% of assets in the sorting process, as assumed for the volatility factor. The size factor, also called Small Minus Big (SMB), was created by dividing the sample into two, with the first half containing the stocks of

companies with lower market value and the second half containing the stocks of larger companies.

We calculated the Sharpe ratio as the annualized difference in the return of portfolio i and the risk-free rate of return (CDI) divided by the annualized volatility of portfolio i .

We can perform the comparison of the Sharpe ratio between portfolios using the test of Jobson & Korkie (1981), adopting the corrections proposed by Memmel (2003). This test shows the statistical significance of the difference in the Sharpe ratio of the portfolios, as given by the following equation:

$$Z = \frac{SR_i - SR_j}{\sqrt{\frac{1}{T} \left(2(1 - \rho_{i,j}) + \frac{1}{2} [SR_i^2 + SR_j^2 - 2SR_i SR_j (1 + \rho_{i,j}^2)] \right)}} \quad (2)$$

In this equation, SR_i refers to the Sharpe ratio of portfolio i , SR_j is the Sharpe ratio of portfolio j , $\rho_{i,j}$ is the correlation between the two portfolios and T is the total number of observations. The calculation of the Sharpe Ratio in this case, as proposed in Jobson & Korkie (1981), involves the mean difference of daily returns between the portfolio i and the risk-free rate divided by the portfolio i daily volatility.

3. RESULTS

Figure 1 shows the plot of each portfolio observed by ranking the volatility of the stocks from IBrX.

Portfolios are plotted by sorting the assets by their volatility. The assets with lower volatility make up portfolio P1 while the assets with higher volatility make up portfolio P3. The blue line represents the Security Market Line, the market benchmark (IBX) being the point where the beta is 1, as well as the risk-free asset when the observed beta is zero. The relationship between portfolio returns and beta is inversely proportional, which is contrary to the expectation given by the CAPM model.

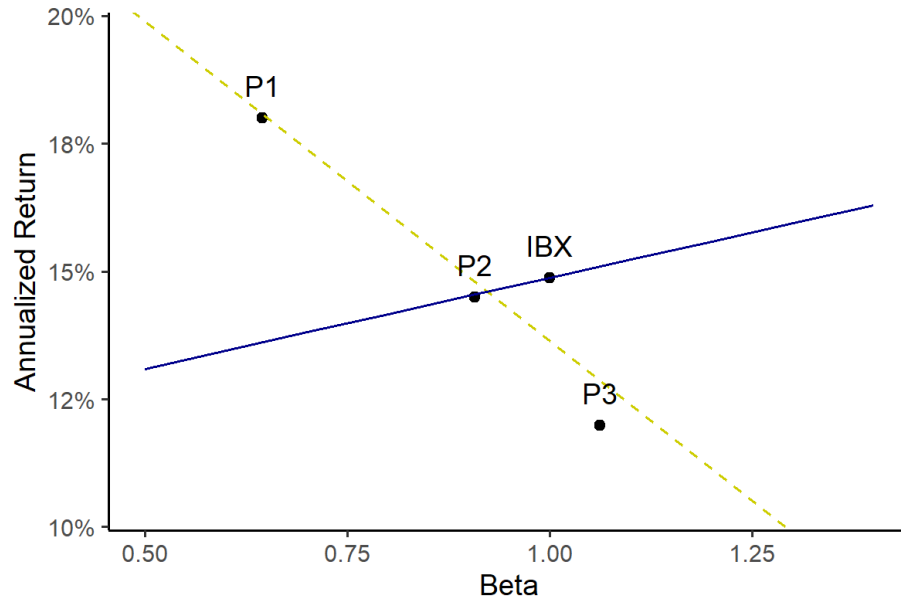


Figure 1 – Portfolios ranked by Volatility

Source: self-made

Plotted in blue is the *Security Market Line* (SML) given by the following equation:

$$r_i = r_f + \beta(R_m - R_f) \quad (3)$$

In this process, the expected return of a portfolio r_i is the sum of the return of the risk-free asset and the market return above the risk-free rate, weighted by the beta of portfolio i . The higher the beta, the higher the return of portfolio i . However, we observed that the portfolios formed from assets with lower volatility are above the SML and have higher returns and lower risk than the portfolios formed from stocks with higher volatility.

We show the results in Table 1. Panel A presents the statistics for each of the three portfolios, as well as a column showing the spread between portfolios P1 and P3 (buy P1 and sell P3, rebalancing every month), which consist of the portfolios with the lowest and highest volatility, respectively. Transaction costs were ignored for all portfolios.

Table 1 – Results of Volatility Portfolios

The first panel (A) of this table presents the results of portfolios ordering stocks from lowest to highest volatility using a 12-month window over the period from January 2003 through December 2021. The return, standard deviation, and Sharpe Index are annualized. The α , β , and $t(\alpha)$ are estimated from the regression for the entire period for each of the deciles. Subsequently, alpha was annualized. Panel B presents, in the first line, the average positive return, followed by the average negative return, and finally the biggest loss in a single day for each portfolio.

	Panel A				IBX
	P1	P2	P3	P1-P3	
Annualized Return (%)	18.01	14.5	11.99	6.02	14.88
Standard Deviation (%)	18.69	25.27	32.22	20.09	26.26
Sharpe Ratio	0.32	0.11	0.02		0.12
z(SR)	1.54	-0.17	-0.59		
Annualized Rp-Rf (%)	6.01	2.86	0.61	5.4	3.2
Beta	0.64	0.91	1.06	-0.42	
Alpha (%)	3.39	0.04	-1.18	4.57	
t(alpha)	1.82	0.02	-0.32	0.2	

	Panel B				IBX
	P1	P2	P3	P1-P3	
Average positive returns (%)	0.83	1.09	1.42	-0.59	1.16
Average negative returns (%)	-0.82	-1.11	-1.47	0.65	-1.17
Biggest loss - day (%)	-11.48	-17.12	-20.13		-14.89

Source: self-made

As observed by Blitz and Van Vliet (2007) in other markets, in Brazil we observed an improvement in absolute return, a reduction in volatility, and consequently an increase in the Sharpe ratio of the P1 portfolio relative to the market benchmark. The SR of the market portfolio was 0.12, while it was 0.32 and 0.02 for the P1 and P3 portfolios, respectively. The difference between the SR of the market and the SR of the P1 portfolio is not statistically significant at the 5% level with a z-statistic of 1.54. We also can see that the alpha of the low (high) volatility portfolio is numerically positive (negative) but not statistically different from 0, considering a 5% significance level.

In Panel B, we analyzed three characteristics of those portfolios. The first and second rows show the average behavior of each portfolio on positive and negative days, respectively. We can observe that the low-risk portfolio tends to perform worse than the high-risk portfolio on days when the return is positive but show more defensive behavior on the days when the return is negative. This panel also shows the biggest loss in a single day that each portfolio has suffered. As with the other metrics, the low-volatility portfolio has a much more conservative behavior than the high-volatility portfolio, which experienced higher losses than the universe of assets.

Strategies based on capturing the spread between higher and lower momentum stocks have extremely negative returns during periods following large stress events and high volatility. Based on these findings by Daniel and Moskowitz (2013), we wanted to test whether the spread

between lower and higher volatility portfolios exhibited similar behavior. To proceed with this analysis, we isolated 2009 from the other years in the sample to examine the behavior of the spread between the lower volatility asset group and the higher volatility asset group.

Table 2 - Time period division

Panel A presents the results of P1, P2, and P3 portfolios from January/2003 to December/2008. The return, volatility, and Sharpe Index are annualized. The α , β , and $t(\alpha)$ are estimated from the regression for the entire period for each of the deciles. Subsequently, alpha was annualized. Panel B presents the results for the year 2009. Finally, Panel C shows the results from January 2010 to December 2021.

Panel A: 2003 - 2008					
	P1	P2	P3	P1-P3	IBX
Annualized Return (%)	27.55	18.84	16.04	11.51	24.96
Standard Deviation (%)	21	28.24	32.5	18.81	30.19
Sharpe Ratio	0.45	0.07	-0.01		0.24
z(SR)	0.88	-1.19	-1.05		
Annualized Rp-Rf (%)	9.49	2.01	-0.4	9.89	7.27
Beta	0.63	0.88	0.92	-0.29	
Alpha (%)	4.03	-4.1	-5.57	9.6	
t(alpha)	1.1	-1.04	-0.82	0.4	

Panel B: 2009					
	P1	P2	P3	P1-P3	IBX
Annualized Return (%)	54.2	88.21	170.4	-116.2	75.16
Standard Deviation (%)	17.03	27.75	31.91	21.19	30.53
Sharpe Ratio	2.35	2.56	4.56		1.93
z(SR)	0.73	1.48	2.49		
Annualized Rp-Rf (%)	40.01	70.91	145.57	-105.56	59.05
Beta	0.48	0.88	0.91	-0.43	
Alpha (%)	11.17	13.55	62.63	-51.46	
t(alpha)	1.2	1.71	3.02	-1.64	

Panel C: 2010 - 2021					
	P1	P2	P3	P1-P3	IBX
Annualized Return (%)	11.01	7.84	2.27	8.74	6.35
Standard Deviation (%)	17.56	23.4	32.07	20.59	23.61
Sharpe Ratio	0.11	-0.04	-0.19		-0.1
z(SR)	1.46	0.59	-0.37		
Annualized Rp-Rf (%)	1.93	-0.97	-6.1	8.03	-2.34
Beta	0.68	0.93	1.2	-0.52	
Alpha (%)	3.22	1.39	-1.61	4.83	
t(alpha)	1.51	0.61	-0.37	0.43	

Source: self-made

In the first panel of Table 2, we analyzed data from the period between 2003 and 2008, being the latter a year in which there was a general deterioration in asset prices.

In Panel B, which includes only data from 2009, a year in which asset prices recovered, we observed worse absolute performance of the low volatility portfolio in comparison with the high volatility portfolio and stock's universe. Although the high volatility portfolio had a negative alpha in regular years (not statistically different from zero), both alpha (statistically significant) and absolute return in a year where prices recovered aggressively show portfolio P3 had a great performance.

Finally, Panel C shows the behavior of the individual portfolios between 2010 to 2021. During this period, despite relevant differences in the annual returns of the individual portfolios, it was not possible to find alphas different from zero using a 5% statistical significance threshold.

We also test these portfolios against other factor models that have more explainability power over the cross-section of expected returns. These models are the Fama & French three-factor model, the Carhart four-factor model, and a 5-factor model comprising the Carhart model and the profitability factor.

Panel A shows the alphas resulting from the regressions under each proposed model, where we can evaluate that if other risk factors are added to the one-factor model, the alphas generated in the portfolios are lower. Panel B presents the t-statistics of the alphas of each regression. As we should expect from the results already presented regarding the CAPM, there are no statistically significant alphas at the 5% confidence interval.

Table 3 – Multifactor regression models

Panel A shows alphas from the regression of the returns of three portfolios sorted by volatility, first using the CAPM model then 3 Factors from Fama & French, then the Cahart (1997) model, and finally a model using 5-factors (Cahart and profitability). The second panel presents the t-statistic for each alpha observed in panel A.

Panel A: Alpha			
	Low.Vol.	Mid.Vol.	High.Vol.
CAPM	3.39	0.04	-1.18
F&F	3.21	-0.41	-2.7
Cahart	2.87	-0.26	-1.92
5 Factors	2.29	-0.74	-1.38

Panel B: t-value			
	Low.Vol.	Mid.Vol.	High.Vol.
CAPM	1.82	0.02	-0.32
F&F	1.76	-0.23	-1.05
Cahart	1.58	-0.14	-0.74

5 Factors 1.27 -0.41 -0.54

Source: self-made

3.1 EVALUATING OTHER FACTORS

Next, we propose the evaluation of 4 different factors: profitability, using the ratio of gross profit to total assets; momentum, observing the last 6 months' total returns; size, ordering stocks by their market capitalization and finally value, using the ratio between market capitalization and book value.

We formed 10 portfolios for each factor. The portfolios with high profitability and high momentum performed better than the market average, while the portfolios with negative exposure to profitability and momentum performed very poorly. Regarding the size and value factors, we found little statistical significance in the relationship between the Sharpe ratio of each portfolio compared to the market portfolio's SR, as seen in table 4. The only portfolio that generated a numerically positive and statistically significant alpha is the winner (momentum) portfolio.

Table 4 – Portfolios ranked by other factors

The first panel presents the results of the portfolios sorted by market capitalization, from the smallest (D1 Portfolio) to the largest companies (D10). Next, Panel B presents the results of the portfolios formed based on the market value of each company in relation to book value. The cheapest stocks using Market Value / Book Value ratio create the D1 portfolio, while the most expensive one forms portfolio D10. Panel C presents the portfolios formed from the ordering of the universe assets based on the absolute return of each stock in the last six months, with monthly rebalancing. Portfolio D1 contains the stocks with the highest momentum (winners), while portfolio D10 contains the lowest (losers). Finally, panel D presents the portfolios formed by ordering by the ratio between gross profit and total assets, where D1 is the portfolio formed by the most profitable stocks. The intermediate portfolios are omitted.

	Panel A: Size							IBX
	D1	D2	D3	D8	D9	D10	D1-D10	
Annualized Return (%)	4.76	14.7	16.49	12.58	10.31	16.15	-11.39	14.88
Standard Deviation (%)	32.39	28.91	28.54	25.07	26.57	29.17	26.04	26.26
Sharpe Ratio	-0.18	0.11	0.16	0.05	-0.03	0.15		0.12
z(SR)	-1.52	-0.02	0.36	-0.63	-1.42	0.63		
Annualized Rp-Rf (%)	-5.89	3.04	4.65	1.14	-0.9	4.35	-10.24	3.2
Beta	0.85	0.87	0.89	0.81	0.9	1.07	-0.22	
Alpha (%)	-6.23	1.48	2.82	-1.05	-3.28	1.45	-7.68	
t(alpha)	-1.19	0.36	0.73	-0.34	-1.2	0.79	-0.92	

Panel B: Value								
	D1	D2	D3	D8	D9	D10	D1-D10	IBX
Annualized Return (%)	12.89	16.86	12.04	8.16	17.31	16.94	-4.05	14.88
Standard Deviation (%)	33.17	30.76	27.76	25.63	24.2	24.89	25.27	26.26
Sharpe Ratio	0.04	0.16	0.02	-0.11	0.22	0.2		0.12
z(SR)	-0.27	0.44	-0.7	-1.93	0.67	0.52		
Annualized Rp-Rf (%)	1.42	4.98	0.65	-2.84	5.38	5.05	-3.63	3.2
Beta	0.95	0.96	0.9	0.84	0.78	0.78	0.17	
Alpha (%)	0.62	3.31	-1.41	-5	3.09	2.94	-2.32	
t(alpha)	0.12	0.8	-0.42	-1.7	1.01	0.88	-0.12	

Panel C: Momentum								
	D1	D2	D3	D8	D9	D10	D1-D10	IBX
Annualized Return (%)	24.86	18.94	20.65	10.27	9.28	-2.34	27.2	14.88
Standard Deviation (%)	26.84	25.32	24.75	28.44	29.71	35.73	30.31	26.26
Sharpe Ratio	0.45	0.27	0.34	-0.03	-0.06	-0.34		0.12
z(SR)	2	1.06	1.67	-1.15	-1.17	-2.49		
Annualized Rp-Rf (%)	12.17	6.85	8.39	-0.94	-1.83	-12.28	24.45	3.2
Beta	0.79	0.81	0.82	0.93	0.92	0.97	-0.18	
Alpha (%)	10.41	4.58	5.9	-2.98	-3.46	-12.26	22.67	
t(alpha)	2.51	1.42	2.01	-0.89	-0.88	-2.25	2.11	

Panel D: Profitability								
	D1	D2	D3	D8	D9	D10	D1-D10	IBX
Annualized Return (%)	21.35	17.53	17.07	8.97	12.29	6.71	14.64	14.88
Standard Deviation (%)	25.38	24.99	25	30.54	30.87	34.47	27.03	26.26
Sharpe Ratio	0.36	0.22	0.21	-0.07	0.03	-0.12		0.12
z(SR)	1.41	0.59	0.5	-1.21	-0.47	-1.2		
Annualized Rp-Rf (%)	9.02	5.58	5.17	-2.11	0.88	-4.14	13.16	3.2
Beta	0.75	0.74	0.75	0.96	0.95	0.97	-0.22	
Alpha (%)	7.13	3.78	3.25	-3.68	-0.59	-4.56	11.69	
t(alpha)	1.87	1.02	0.9	-0.93	-0.14	-0.87	1.02	

Source: self-made

When comparing portfolios' performance, we noticed that portfolios that presented a higher return than the market portfolio, on average, had a beta smaller than one, while the portfolios that had a performance lower than the market portfolio presented betas greater than one, i.e., a reduction in portfolio risk seems tied to superior performance in terms of absolute return, which results in a better risk-return ratio than the initial situation. Although it's clear that the relationship exists, we can't say that there is causality.

3.2 DOUBLE SORTING

After sorting the portfolios by a primary characteristic, whose results we reflect in table 4, we performed a double sorting process. First, we separated the stocks into groups according to the primary factor (size, value, momentum, and profitability) following the same process of creating the risk factors used to test the portfolios (SMB, HML, WML, and PMU, respectively). Next, within each subgroup, we ranked the stocks by the 12-month volatility of each stock, forming new deciles, whose results we show in table 5.

When comparing each portfolio formed from the simple ordering with the equivalent portfolio from the double sorting process, it is possible to notice an improvement in terms of risk and return of the portfolios that receive the stocks with lower volatility. There was an improvement in the portfolios' Sharpe ratio when compared to the market portfolio, as well as in some portfolios' alpha using the CAPM model.

Looking specifically at the portfolio of the first decile formed by value and then by low volatility, for example, it was possible to have a bigger return than those observed in simple ordering portfolios with lower volatility. This translates into a portfolio whose Sharpe Ratio is four times bigger (0.04 to 0.25).

Table 5 – Double sorted portfolios

This table presents the results obtained by double sorting stocks, first according to a specific characteristic such as size, value, momentum, and profitability, and then by volatility. Panel A reflects the results obtained from sorting by size and volatility. Firstly, the universe was divided into two parts, as was the process of forming the SMB factor. Next, each half was sorted by volatility, with the D1 portfolios in each half having the lowest volatility stocks. In the first D1, the firms are small and have low volatility. In the next D1, the firms are larger and exhibit low volatility. Panels B, C, and D deal with the results for value, momentum, and quality. In all three panels, the data are first divided into three large groups according to the formation process of each factor (HML, UMD, and QMJ, respectively) and then sorted by volatility.

Panel A: Size							
	30% Small			30% Big			IBX
	D1	D2	D3	D3	D4	D5	
Annualized Return (%)	16.2	9.27	9.55	15.71	12.48	9.99	14.88
Standard Deviation (%)	21.91	29.8	38.01	21.15	26.8	32.29	26.26
Sharpe Ratio	0.2	-0.06	-0.04	0.19	0.04	-0.04	0.12
z(SR)	0.36	-1.1	-0.6	0.33	-0.95	-1.28	
Annualized Rp-Rf (%)	4.39	-1.84	-1.59	3.95	1.04	-1.2	3.2
Beta	0.66	0.9	1.06	0.71	0.96	1.12	
Alpha (%)	2.4	-3.27	-1.31	1.44	-1.66	-3.31	
t(alpha)	0.76	-0.79	-0.22	0.61	-0.77	-1.08	

Panel B: Value

	30% Value			30% Growth			IBX
	D1	D2	D3	D1	D2	D3	
Annualized Return (%)	17.92	16.92	5.6	14.85	15.02	13.1	14.88
Standard Deviation (%)	23.33	31.34	38.02	19.82	25.55	30.7	26.26
Sharpe Ratio	0.25	0.16	-0.14	0.16	0.13	0.05	0.12
z(SR)	0.81	0.48	-1.2	0.04	0.04	-0.33	
Annualized Rp-Rf (%)	5.94	5.03	-5.14	3.17	3.33	1.6	3.2
Beta	0.73	1.01	1.08	0.63	0.84	0.96	
Alpha (%)	3.73	3.22	-5	0.94	1.03	-0.02	
t(alpha)	1.19	0.82	-0.87	0.37	0.34	-0.01	

Panel C: Momentum							
	30% Winners			30% Losers			IBX
	D1	D2	D3	D1	D2	D3	
Annualized Return (%)	22.19	18.77	22.36	7.7	3.84	2.91	14.88
Standard Deviation (%)	20.67	25.85	31.08	24.71	30.69	39.09	26.26
Sharpe Ratio	0.47	0.26	0.32	-0.13	-0.22	-0.19	0.12
z(SR)	2.2	1	1.46	-1.92	-2.35	-1.51	
Annualized Rp-Rf (%)	9.78	6.7	9.93	-3.25	-6.72	-7.56	3.2
Beta	0.65	0.83	0.95	0.78	0.96	1.09	
Alpha (%)	7.47	4.47	8.35	-5.25	-8.22	-7.11	
t(alpha)	2.64	1.36	1.89	-1.68	-2.12	-1.19	

Panel D: Profitability							
	30% Profitable			30% Unprofitable			IBX
	D1	D2	D3	D1	D2	D3	
Annualized Return (%)	17.77	17.55	19.57	12.29	10.4	3.31	14.88
Standard Deviation (%)	19.25	25.46	31.43	24.34	32.3	40.44	26.26
Sharpe Ratio	0.3	0.22	0.24	0.04	-0.03	-0.18	0.12
z(SR)	0.9	0.62	0.85	-0.69	-0.8	-1.38	
Annualized Rp-Rf (%)	5.8	5.61	7.42	0.87	-0.82	-7.19	3.2
Beta	0.57	0.78	0.92	0.77	0.99	1.13	
Alpha (%)	3.78	3.66	6.27	-1.21	-2.12	-6.48	
t(alpha)	1.34	1.02	1.3	-0.38	-0.49	-1.05	

Source: self-made

To test, in a more systematic way, whether there was a statistically significant improvement after the double sorting process, we used the Jobson & Korkie test again, but in this case, comparing the SR of the portfolios observed by simple sorting with the SR observed by double ordering. We can see the results in table 6.

Within the size factor, the process that ranks first by size and then by volatility presented statistical significance in improving the Sharpe Ratio in the first portfolio (D1) and a numerical improvement in 5 of the remaining 9 portfolios.

Within the value factor, we first sorted stocks by the price-to-book relation, forming a group of the 30% cheapest stocks, then ranked by volatility. In this case, there were no statistically significant SR improvements. The results are similar for the profitability factor.

For momentum, there was a statistically significant improvement in the last portfolio (D10), which is unexpected since this portfolio is composed of companies with low momentum and high volatility. This may be explained by fact that the momentum D10 simple sorted portfolio had a very low Sharpe Ratio (-0.34), so the improvement was big, but it's still the double sorted portfolio with the lowest Sharpe Ratio (-0.18). In the other portfolios, there was no statistically significant improvement.

Blitz and Van Vliet (2007) observed that the volatility reduction by this method was greater than that found in the portfolios generated by the minimum variance process in American market portfolios, developed by Clarke, de Silva and Thorley (2006). In conformity with this study, we were able to see an expressive reduction in the volatility of the portfolios that have high exposure to the factors (SMB, HML, WML, PMU) and high exposure to the low volatility factor (D1 portfolios). We were also able to see a numerical improvement in the Sharpe Ratios, although not statistically significant, except in the case of the size portfolio.

Table 6 – SR Z-Score

This table shows the z-score of the comparison between the Sharpe Ratios (SR) of portfolios ordered by a single characteristic (size, value, momentum, and quality) with the SR of the portfolios created by double sorting. Initially, the assets were sorted based on a primary characteristic of the firms. Next, they were sorted according to their 12-month volatility. Finally, the SR of the single sorted portfolios was compared to the respective SR of the double sorted portfolios. The z-scores with statistical significance are shown in bold.

Panel A: Size		Panel B: Value	
	<u>Dn</u>		<u>Dn</u>
D1	2.05	D1	1
D2	-1.32	D2	0.01
D3	-1.07	D3	-0.73
D4	0.5	D4	1.7
D5	0.16	D5	-1.47
D6	0.07	D6	-1.39
D7	-0.6	D7	1.06
D8	0.86	D8	1.7
D9	0.67	D9	-0.69
D10	-1.43	D10	-0.98

Panel C: Momentum		Panel D: Quality	
	<u>Dn</u>		<u>Dn</u>
D1	-0.06	D1	-0.49

D2	-0.07	D2	-0.01
D3	0.09	D3	0.4
D4	0.88	D4	1.29
D5	-1.13	D5	-0.59
D6	-0.12	D6	-0.76
D7	0.93	D7	0.1
D8	-1.03	D8	0.56
D9	-1.39	D9	-0.36
D10	2.03	D10	-0.29

Source: self-made

3.3 TRADING STRATEGY

Based on the data, the proposal was to develop a strategy that goes long in 30% of stocks with lower volatility and short 30% of stocks with higher volatility, allocation of all cash by buying stocks with lower volatility and getting back part of this cash by selling stocks with higher volatility. The equation below gives the short exposure percentage.

$$\text{Short exposure (\%)} = \frac{\beta_{\text{Low Volatility}}}{\beta_{\text{High Volatility}}} \quad (4)$$

We did not consider the borrowing cost in this study, despite considering risk-free returns from the cash raised by short selling.

We tested the model using both the pure historical beta and the beta coefficient adjusted by the method of Blume (1975) in the proportion of two-thirds for the historical beta of one year and one-third converging to one, according to the equation:

$$\beta_{\text{adjusted}} = \frac{2}{3} * \beta_{MQO} + \frac{1}{3} * 1 \quad (5)$$

For comparison reasons we also considered a simple Long & Short strategy that buys 100% of the low volatility portfolio, sells 100% of the high volatility portfolio and uses the available cash to go long in the risk-free rate since the Long & Short is dollar neutral.

The strategy that used the one-year beta presented an annualized return of 14.95% with a standard deviation of 11.49%. We observed a Sharpe ratio of 0.34 while the maximum drawdown was 30.08%. Blume's model strategy had a 13.7% annualized return, a standard

deviation of 13.46%, and a 0.2 Sharpe ratio. The simple Long & Short strategy had a lower return and a higher standard deviation, which implies a significantly smaller Sharpe Ratio. The results are presented in Table 7.

Table 7 – Strategy metrics

In this table are the results of a strategy that goes long low volatility portfolio and short high volatility portfolio. The main objective was to reach a zero-beta portfolio. The first process used the historical beta to create a market-neutral strategy, while the second process had the same objective following the Blume corrections. The third process is a simple Long & Short dollar neutral portfolio that uses the available cash to go long at the risk-free rate.

	Panel A		
	Pure Beta	Beta Blume	Simple LS
Annualized Return (%)	14.95	13.7	10.11
Standard Deviation (%)	11.49	13.46	19.94
Sharpe Ratio	0.34	0.2	-0.03
z(SR)	0.61	0.23	-0.26
Annualized Rp-Rf (%)	3.87	2.74	-0.51
Beta	-0.02	-0.13	-0.36
Alpha (%)	4.65	4.31	3.29
t(alpha)	1.66	1.36	0.78
Max. Drawdown	30.08	48.07	77.99

Source: self-made

4. POSSIBLE CAUSES OF VOLATILITY EFFECT

The CAPM model assumes a positive relationship between risk and return, i.e., the higher the beta of a stock, the higher the expected returns.

Despite the positive relationship proposed by the CAPM, we observed a negative risk-return relationship in the portfolios ranked by volatility, defined by the yellow curve plotted in figure 1, given by a regression of the portfolios data, using the ordinary least squares method.

The persistence of the positive returns from this anomaly may be related to the fact that the benchmark where we perform the screenings is a market index. Purchasing low volatility stocks, with betas smaller than one, to the detriment of higher volatility stocks, which have a beta higher closer to one, would create a portfolio that has a higher tracking error relative to the benchmark, causing managers of stock portfolios that follow a market index to hold more assets with a beta close to one.

Using low volatility stocks to form a portfolio would require leverage to match the risk of this portfolio to benchmark beta. Black (1972) had already identified the restrictions on asset

lending and leverage as an argument for the superior performance of low beta assets. In a more recent paper, Blitz Falkenstein and Van Vliet (2014), analyzed the causes of the volatility effect. They related one point discussed in the article to the freedom that the CAPM model gives in the process of leverage or short selling. As assessed by the authors, in the real world, investors have limits on leverage through mandates.

Another premise of the CAPM model that was also discussed by the same authors is the fact that the model has no transactional costs. The evidence shows that stocks with higher borrowing costs are the ones with the greatest divergence from the model pricing. The explanation for the higher returns of low volatility stocks may be because of investor behavior, seeking higher returns by allocating to stocks with higher betas, in the belief that the CAPM is sustainable, even partially.

In summary, the limitations to leverage, aligned with higher borrowing costs for riskier assets and the search for more profitable portfolios through allocation to higher volatility stocks result in lower demand for low volatility stocks that may explain part of this anomaly resulting in an inverse relationship between the risk measured by beta and the return of portfolios as observed in figure 1.

5. CONCLUSION

Low volatility anomaly has been extensively documented in academic studies. The behavior pattern considered as one of the basic pillars of finance assumes that the higher the risk, the higher the return. In this study, it was possible to verify that return was inversely proportional to the risk of the assets. The results observed in the local market are similar, although in a smaller magnitude, to those observed by research developed in other countries around the world, especially in the article by Blitz and Van Vliet (2007).

The portfolios formed from ordering by stocks' volatility presented an inverse relationship to CAPM model expectation. The 30% stocks with lower volatility presented superior performance, measured by absolute return and SR when compared to the stocks' universe and the portfolio formed by the assets with higher volatility. As seen previously, this result was consistent not only with the results presented by Blitz and Van Vliet (2007) in a study carried out for North American, European, and Asian countries but also with the results of Ang et al. (2006) who showed negative alpha in American stocks with high idiosyncratic volatility, analyzing a 1-year volatility history.

Under the CAPM model, we observed no alpha on these portfolios, but we were able to see a bigger Sharpe Ratio of the low volatility portfolio, although with no statistical significance when compared to the Sharpe Ratio of the market portfolio.

It was also possible to use the volatility sorting procedure in a dual sort model, numerically improving the generation of alpha and portfolios' SR pre-sorted by other risk factors in some cases, despite not being able to find statistical significance in all of them.

With the data observed, we suggested a strategy that could deliver a better risk-return relationship. We structured the operation to minimize or even have zero exposure to market beta, generating results that were independent of the market's behavior during the analyzed period. We did not reflect transactional costs on our results, which may be the subject of another study that addresses how the dynamics of trading and borrowing costs would affect the results observed, thus allowing an evaluation of the economic viability of implementing this strategy in an investment vehicle.

Finally, we investigated explanations to support the results observed and to support this anomaly since the first academic evidence. Although several hypotheses are discussed in the literature, we believe that the limitations to leverage by investors are linked to the high borrowing costs of high volatility stocks and investors' behavior towards buying higher volatility stocks to increase portfolio returns by capturing a higher equity risk premium are directly linked to the low volatility portfolios risk-return relationship.

As noted by Blitz and Van Vliet, exploiting the volatility effect is difficult for managers with mandates to follow a market benchmark and who cannot or do not want to leverage their portfolios or do short selling. Investors who seek to capture alphas or build strategies with higher-than-market SR can benefit from this anomaly in their portfolio management.

6. REFERENCES

- Blitz, David C. e Vliet, Pim van. (2007). The Volatility Effect: Lower Risk without Lower Return. *Journal of Portfolio Management*, Fall, Páginas 102-113;
- Markowitz, Harry. "The utility of wealth." *Journal of political Economy* 60.2 (1952): 151-158.
- Blume, Marshall E. (1975). Betas and Their Regression Tendencies. *The Journal of Finance*, Volume 30, Edição 3, Páginas 785-795
- Asness, Clifford, and Andrea Frazzini. "The devil in HML's details." *The Journal of Portfolio Management* 39.4 (2013): 49-68.
- Fama E.F. e K.R. French. (1993) The Cross-Section of Expected Stock Returns. *Journal of Finance*, Volume 47, Número 2, Páginas 427-446
- Jegadeesh N. e S. Titman. (1993). Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. *Journal of Finance*, 48(1), Páginas. 65-91
- Rouwenhorst, K. (1998). International Momentum Strategies. *Journal of Finance*, Volume 53, Edição 1, Páginas 267-284
- Novy-Marx, Robert. "The other side of value: The gross profitability premium." *Journal of financial economics* 108.1 (2013): 1-28.
- Jobson, J. Dave, and Bob M. Korkie. "Performance hypothesis testing with the Sharpe and Treynor measures." *Journal of Finance* (1981): 889-908.
- Memmel, Christoph. "Performance hypothesis testing with the Sharpe ratio." Available at SSRN 412588 (2003).
- Daniel, Kent D., and Tobias J. Moskowitz. "Momentum crashes." *Swiss Finance Institute Research Paper* 13-61 (2013): 14-6.
- Clarke, R., de Silva, H., e Thorley, S. (2006) Minimum-Variance Portfolios in the U.S. Equity Market. *Journal of Portfolio Management*, Volume 33, Número 1, Páginas 10-24
- Black, Fischer. "Capital market equilibrium with restricted borrowing." *The Journal of business* 45.3 (1972): 444-455.

Blitz, David, Eric Falkenstein, and Pim Van Vliet. "Explanations for the volatility effect: An overview based on the CAPM assumptions." *The Journal of Portfolio Management* 40.3 (2014): 61-76.

Ang A., R.J. Hodrick, Y. Xing and X. Zhang. (2006). The Cross-Section of Volatility and Expected Returns. *Journal of Finance*, 61, Páginas. 259-299

Carhart, Mark M. (1997). On Persistence in Mutual Fund Performance. *The Journal of Finance*, Volume 52, Número 1, Páginas 57-82

7. ONLINE SUPPLEMENTAL MATERIAL

The datasets and code used to produce the tables can be accessed via the GitHub link:

<https://github.com/pedroteles17/Paper-Avant>